

### HOMEWORK 3

**Problem 1** Solve the congruence

$$x^2 + 2 \equiv 0 \pmod{1759}.$$

**Problem 2** Solve the congruence

$$x^2 + 3 \equiv 0 \pmod{1399}.$$

**Problem 3** Solve the congruence

$$x^2 + 1 \equiv 0 \pmod{p}, \quad \text{where } p = 4n + 1.$$

**Problem 4** Let  $M > 2$  satisfy  $M \equiv 1 \pmod{2}$  and  $(a, M) = 1$ . Show that

$$\left| \sum_{n=0}^{M-1} e\left(\frac{an^2}{M}\right) \right|^2 = M, \quad \text{where } e(x) = e^{2\pi i x}.$$

**Problem 5** Use previous problem to show that for prime  $p > 2$ ,

$$\left| \sum_{n=1}^{p-1} \binom{n}{p} e\left(\frac{an}{p}\right) \right| = \sqrt{p},$$

where  $(a, p) = 1$ .

**Problem 6** Solve the congruence

$$7x^4 + 19x + 25 \equiv 0 \pmod{27}.$$

**Problem 7** Solve the congruence

$$9x^2 + 29x + 61 \equiv 0 \pmod{64}.$$